Fourier series synthesis

$$x(t) = \sum_{k=-\infty}^{\infty} a_k e^{j \frac{2\pi}{T_0} k t}$$

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Multiply both sides by $e^{-j \frac{2\pi}{T_0} l t}$

and integrate over the period T_0

$$\int_0^{T_0} x(t) e^{-j \frac{2\pi}{T_0} l t} dt = \int_0^{T_0} \left(\sum_{k=-\infty}^{\infty} a_k e^{+j \frac{2\pi}{T_0} k t} \right) e^{-j \frac{2\pi}{T_0} l t} dt$$

$$= \sum_{k=-\infty}^{\infty} a_k \left(\int_0^{T_0} e^{+j \frac{2\pi}{T_0} k t} e^{-j \frac{2\pi}{T_0} l t} dt \right)$$

$$= \sum_{k=-\infty}^{\infty} a_k \left(\int_0^{T_0} e^{j \frac{2\pi}{T_0} (k-l) t} dt \right) = a_l T_0$$

Two cases:

$k = l$: $\int_0^{T_0} e^{j \frac{2\pi}{T_0} (0) t} dt = \int_0^{T_0} 1 \, dt = T_0$

$k \neq l$: $\int_0^{T_0} e^{j \frac{2\pi}{T_0} m t} dt = \left. \frac{e^{j \frac{2\pi}{T_0} m t}}{j \frac{2\pi}{T_0} m} \right|_0^{T_0} = \frac{e^{j \frac{2\pi}{T_0} m T_0} - 1}{j \frac{2\pi}{T_0} m} = 0$

let $m = k - l$.

This yields $a_l = \frac{1}{T_0} \int_0^{T_0} x(t) e^{-j \frac{2\pi}{T_0} l t} dt$

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